Converting Depth Images and Point Clouds for Feature-based Pose Estimation – Supplementary Material

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I. Derivation of the Bearing Angle formula based on the cosine theorem

This section provides the exact derivation of the Bearing Angle (BA) formula. The literature on BA disagrees on the formula (see [1] versus [2]). The goal of the full chain of derivation and graphical presentation is to provide a trustworthy and checkable background for the provided formula.

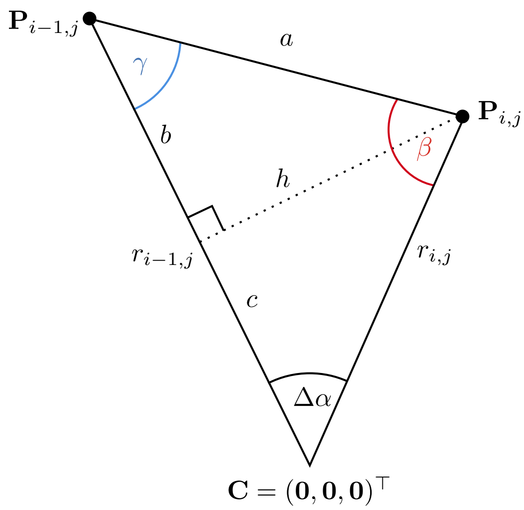

Fig. 1. Derivation of the BA with all support variables. Deriving the Bearing Angle requires additional support variables that are introduced with this figure. Using fundamental trigonometry allows derivation of the formula. Compare Fig. 2 of the paper.

The cosine theorem holds true for all triangles and is the central equality for the derivation.

Ansatz:

$$r_{i-1,j}^2 = r_{i,j}^2 + a^2 - 2 r_{i,j} a \cos(\beta_{i,j}) \qquad \text{(Cosine Theorem)} \tag{1}$$

Introducing supporting variables at the positions given in Figure 1 allows substituting the missing quantity a.

$$h = r_{i,j} \sin(\Delta\alpha_{i,j}) \tag{2}$$
$$c = r_{i,j} \cos(\Delta\alpha_{i,j}) \tag{3}$$
$$b = r_{i-1,j} - r_{i,j} \cos(\Delta\alpha_{i,j}) \tag{4}$$
$$a^2 = h^2 + b^2 \tag{5}$$
$$a^2 = \underline{(r_{i,j} \sin(\Delta\alpha_{i,j}))^2} + \underline{(r_{i-1,j} - r_{i,j} \cos(\Delta\alpha_{i,j}))^2} \tag{6}$$
$$a^2 = \underline{r_{i,j}^2 (\sin(\Delta\alpha_{i,j}))^2} + \underline{r_{i-1,j}^2 - 2 r_{i-1,j} r_{i,j} \cos(\Delta\alpha_{i,j})} + \underline{r_{i,j}^2 (\cos(\Delta\alpha_{i,j}))^2} \tag{7}$$
$$a^2 = \underline{r_{i,j}^2 (\sin(\Delta\alpha_{i,j}))^2 + r_{i,j}^2 (\cos(\Delta\alpha_{i,j}))^2} + \underline{r_{i-1,j}^2 - 2 r_{i-1,j} r_{i,j} \cos(\Delta\alpha_{i,j})} \tag{8}$$
$$a^2 = \underline{r_{i,j}^2} + \underline{r_{i-1,j}^2 - 2 r_{i-1,j} r_{i,j} \cos(\Delta\alpha_{i,j})} \tag{9}$$

For a holds the condition $a > 0$ for all values, because the constraints $r_{i,j} > 0$, $r_{i-1,j} > 0$ and $0 < \Delta\alpha_{i,j} < \pi$ result from the properties of depth sensors.

The variable a is now substituted in the ansatz. Consequent simplification of the equations yields the final formula for the bearing angle.

$$r_{i-1,j}^2 = r_{i,j}^2 + a^2 - 2r_{i,j}a\cos(\beta_{i,j}) \tag{10}$$

$$\underline{r_{i-1,j}^2} = \underline{r_{i,j}^2} + \underline{r_{i,j}^2} + \underline{r_{i-1,j}^2} - 2r_{i-1,j}r_{i,j}\cos(\Delta\alpha_{i,j}) - 2r_{i,j}\sqrt{r_{i,j}^2 + r_{i-1,j}^2 - 2r_{i-1,j}r_{i,j}\cos(\Delta\alpha_{i,j})}\cos(\beta_{i,j}) \tag{11}$$

$$\underline{0} = \underline{2}\underline{r_{i,j}^2} + \underline{0} - \underline{2}r_{i-1,j}\underline{r_{i,j}}\cos(\Delta\alpha_{i,j}) - \underline{2}\underline{r_{i,j}}\sqrt{r_{i,j}^2 + r_{i-1,j}^2 - 2r_{i-1,j}r_{i,j}\cos(\Delta\alpha_{i,j})}\cos(\beta_{i,j}) \tag{12}$$

$$0 = \underline{r_{i,j}^2} - r_{i-1,j}\underline{r_{i,j}}\cos(\Delta\alpha_{i,j}) - \underline{r_{i,j}}\sqrt{r_{i,j}^2 + r_{i-1,j}^2 - 2r_{i-1,j}r_{i,j}\cos(\Delta\alpha_{i,j})}\cos(\beta_{i,j}) \tag{13}$$

$$0 = \underline{r_{i,j}} - \underline{1}r_{i-1,j}\cos(\Delta\alpha_{i,j}) - \underline{1}\sqrt{r_{i,j}^2 + r_{i-1,j}^2 - 2r_{i-1,j}r_{i,j}\cos(\Delta\alpha_{i,j})}\cos(\beta_{i,j}) \tag{14}$$

$$\cos(\beta_{i,j}) = \frac{r_{i,j} - r_{i-1,j}\cos(\Delta\alpha_{i,j})}{\sqrt{r_{i,j}^2 + r_{i-1,j}^2 - 2r_{i-1,j}r_{i,j}\cos(\Delta\alpha_{i,j})}} \tag{15}$$

$$\beta_{i,j} = \arccos\left(\frac{r_{i,j} - r_{i-1,j}\cos(\Delta\alpha_{i,j})}{\sqrt{r_{i,j}^2 + r_{i-1,j}^2 - 2r_{i-1,j}r_{i,j}\cos(\Delta\alpha_{i,j})}}\right) \tag{16}$$

II. Transformation of Bearing Angle formula into calculation of an angle between two vectors

Subsequently, we want to demonstrate that the original formulation of the BA from Harati *et al*. [1] (which was derived from the cosine theorem in Sec. I) can be transformed to the calculation of an angle between two vectors. The original formula is

$$\beta_{i,j} = \arccos\left(\frac{r_{i,j} - r_{i-1,j}\cos(\Delta\alpha_{i,j})}{\sqrt{r_{i,j}^2 + r_{i-1,j}^2 - 2r_{i,j}r_{i-1,j}\cos(\Delta\alpha_{i,j})}}\right) \tag{17}$$

$$\Delta\alpha_{i,j} = \arccos\left(\frac{\overrightarrow{\mathbf{P}_{i,j}} \cdot \overrightarrow{\mathbf{P}_{i-1,j}}}{\|\mathbf{P}_{i,j}\|_2\|\mathbf{P}_{i-1,j}\|_2}\right), \tag{18}$$

with Bearing Angle $\beta_{i,j}$ of 3D point $\mathbf{P}_{i,j}$with distance $r_{i,j} = \|\mathbf{P}_{i,j}\|_2$ from the sensor center, point $\mathbf{P}_{i-1,j}$with distance $r_{i-1,j} = \|\mathbf{P}_{i-1,j}\|_2$ from the sensor center, and angular resolution $\Delta\alpha_{i,j}$ between points $\mathbf{P}_{i,j}$and $\mathbf{P}_{i-1,j}$.

In the following, we replace a dot product with vector multiplication: $\vec{\mathbf{P}} \cdot \vec{\mathbf{P}}$ becomes $\mathbf{P}^\top\mathbf{P}$.

$$\cos(\beta_{i,j}) = \frac{\|\mathbf{P}_{i,j}\|_2 - \|\mathbf{P}_{i-1,j}\|_2\cos(\Delta\alpha_{i,j})}{\sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\|\mathbf{P}_{i,j}\|_2\|\mathbf{P}_{i-1,j}\|_2\cos(\Delta\alpha_{i,j})}} \tag{19}$$

$$\Delta\alpha_{i,j} = \frac{\mathbf{P}_{i,j}^\top\mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2\|\mathbf{P}_{i-1,j}\|_2} \tag{20}$$

$$\cos(\beta_{i,j}) = \frac{\|\mathbf{P}_{i,j}\|_2 - \|\mathbf{P}_{i-1,j}\|_2 \frac{\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2}}{\sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2 \cos(\Delta\alpha_{i,j})}} \tag{21}$$

$$\cos(\beta_{i,j}) = \frac{\|\mathbf{P}_{i,j}\|_2 - {\color{red}\|\mathbf{P}_{i-1,j}\|_2} \frac{\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 {\color{red}\|\mathbf{P}_{i-1,j}\|_2}}}{\sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2 \cos(\Delta\alpha_{i,j})}} \tag{22}$$

$$\cos(\beta_{i,j}) = \frac{\|\mathbf{P}_{i,j}\|_2 - \frac{\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2}}{\sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2 \cos(\Delta\alpha_{i,j})}} \tag{23}$$

$$\cos(\beta_{i,j}) = \frac{\|\mathbf{P}_{i,j}\|_2 {\color{green}\frac{\|\mathbf{P}_{i,j}\|_2}{\|\mathbf{P}_{i,j}\|_2}} - \frac{\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2}}{\sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2 \cos(\Delta\alpha_{i,j})}} \tag{24}$$

$$\cos(\beta_{i,j}) = \frac{\frac{\|\mathbf{P}_{i,j}\|_2^2 - \mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2}}{\sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2 \cos(\Delta\alpha_{i,j})}} \tag{25}$$

$$\cos(\beta_{i,j}) = \frac{\|\mathbf{P}_{i,j}\|_2^2 - \mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 \sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2 \cos({\color{green}\Delta\alpha_{i,j}})}} \tag{26}$$

$$\cos(\beta_{i,j}) = \frac{\|\mathbf{P}_{i,j}\|_2^2 - \mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 \sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2 \frac{\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2}}} \tag{27}$$

$$\cos(\beta_{i,j}) = \frac{\|\mathbf{P}_{i,j}\|_2^2 - \mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 \sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2{\color{red}\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2} \frac{\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{{\color{red}\|\mathbf{P}_{i,j}\|_2 \|\mathbf{P}_{i-1,j}\|_2}}}} \tag{28}$$

$$\cos(\beta_{i,j}) = \frac{{\color{green}\|\mathbf{P}_{i,j}\|_2^2} - \mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 \sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}} \tag{29}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top \mathbf{P}_{i,j} - \mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 \sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}} \tag{30}$$

$$\cos(\beta_{i,j}) = \frac{{\color{blue}\mathbf{P}_{i,j}^\top} \mathbf{P}_{i,j} - {\color{blue}\mathbf{P}_{i,j}^\top} \mathbf{P}_{i-1,j}}{\|\mathbf{P}_{i,j}\|_2 \sqrt{\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2 - 2\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}} \tag{31}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top (\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2 \sqrt{{\color{green}\|\mathbf{P}_{i,j}\|_2^2 + \|\mathbf{P}_{i-1,j}\|_2^2} - 2\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}} \tag{32}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top (\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2 \sqrt{\mathbf{P}_{i,j}^\top \mathbf{P}_{i,j} + \mathbf{P}_{i-1,j}^\top \mathbf{P}_{i-1,j} - 2\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j}}} \tag{33}$$

$$\text{replace:} \ -2\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j} \ \text{with} \ -\mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j} - \mathbf{P}_{i-1,j}^\top \mathbf{P}_{i,j} \tag{34}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top (\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2 \sqrt{\mathbf{P}_{i,j}^\top \mathbf{P}_{i,j} + \mathbf{P}_{i-1,j}^\top \mathbf{P}_{i-1,j} - \mathbf{P}_{i,j}^\top \mathbf{P}_{i-1,j} - \mathbf{P}_{i-1,j}^\top \mathbf{P}_{i,j}}} \tag{35}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2\sqrt{\mathbf{P}_{i,j}^\top\mathbf{P}_{i,j} + \mathbf{P}_{i-1,j}^\top\mathbf{P}_{i-1,j} - \mathbf{P}_{i,j}^\top\mathbf{P}_{i-1,j} - \mathbf{P}_{i-1,j}^\top\mathbf{P}_{i,j}}} \tag{36}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2\sqrt{\mathbf{P}_{i,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j}) + \mathbf{P}_{i-1,j}^\top(\mathbf{P}_{i-1,j} - \mathbf{P}_{i,j})}} \tag{37}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2\sqrt{\mathbf{P}_{i,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j}) - \mathbf{P}_{i-1,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}} \tag{38}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2\sqrt{(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}} \tag{39}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2\sqrt{\|\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j}\|_2^2}} \tag{40}$$

$$\cos(\beta_{i,j}) = \frac{\mathbf{P}_{i,j}^\top(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}{\|\mathbf{P}_{i,j}\|_2\|\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j}\|_2} \qquad \text{Compare Eq. (9) of the paper} \tag{41}$$

$$\cos(\beta_{i,j}) = \frac{\overrightarrow{\mathbf{P}_{i,j}} \cdot \overrightarrow{(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}}{\left|\overrightarrow{\mathbf{P}_{i,j}}\right| \left|\overrightarrow{(\mathbf{P}_{i,j} - \mathbf{P}_{i-1,j})}\right|} \tag{42}$$

$$\cos(\beta_{i,j}) = \cos\sphericalangle(\overrightarrow{\mathbf{P}_{i,j}}, \overrightarrow{\mathbf{P}_{i-1,j}}) \tag{43}$$

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